Tentamen: Introduction to Plasma Physics

January 30, 2012

13.30-16.30 h

Please write clearly your name and student number on *each* sheet. You can use either Dutch or English. A list of plasma formulas is attached.

PROBLEM 1 (20 points)

An infinite plate is immersed in a macroscopically neutral plasma consisting of mobile electrons and fixed, singly-charged ions. The plasma has a number density n_0 and temperature T. The plate is located in the x, y plane and is held at a potential V_0 relative to zero potential at a large distance from the plate.

a. Use Poisson's law, $\nabla^2\Phi(z) = -\rho/\epsilon_0$ to show that the potential $\Phi(z)$ satisfies

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}z^2} = \frac{en_0}{\epsilon_0}\left[\exp\left(e\Phi/k_BT\right) - 1\right] \tag{1}$$

with e the unit of charge and k_B Boltzmann's constant.

b. Solve equation (1) assuming that $e\Phi \ll k_BT$ and give a physical interpretation.

c. Discuss how the value of the Debye length λ_D is relevant for the question whether an ionized gas qualifies as a plasma.

PROBLEM 2 (20 points)

An electron lies at rest in the magnetic field of an infinite straight wire along the z-axis carrying a current I in the $+z$ direction. The wire suddenly acquires at time $t = 0$ a linear charge density $\lambda_0 > 0$ without affecting I. The electron gains energy from the electric field and begins to drift.

a. Draw a diagram showing the directions of I, the electric field $\mathbf{E}$, the magnetic field $\mathbf{B}$, the $\mathbf{E}\times\mathbf{B}$ drift $\mathbf{v_E}$, the ∇B drift $\mathbf{v}_{\nabla\mathbf{B}}$ and the curvature drift $\mathbf{v}_{cB}$. Also draw schematically the orbit of the electron.

b. Calculate the electric and magnetic fields $E(r)$ and $B(r)$ as a function of the radial distance r from the wire.

c. Derive formulas for the $\mathbf{E}\times\mathbf{B}$ and ∇B drifts.

PROBLEM 3 (25 points)

Consider a cylindrically symmetric plasma with radius a confined in a theta pinch as shown in Figure 1. The radius of the plasma chamber wall is b with $b > a$. A purely azimuthal current I in a cylindrical coil produces a uniform axial vacuum magnetic field B_0 in the region between the plasma and the cylindrical plasma chamber wall $a < r < b$. The induced current density in the plasma is in the azimuthal direction and given by $J_\theta(r) = 6\,r(r-a)$, the resulting axial magnetic field in the plasma column is $B_z(r)$. The radial plasma pressure distribution is $p(r)$ with $p(a) = 0$.

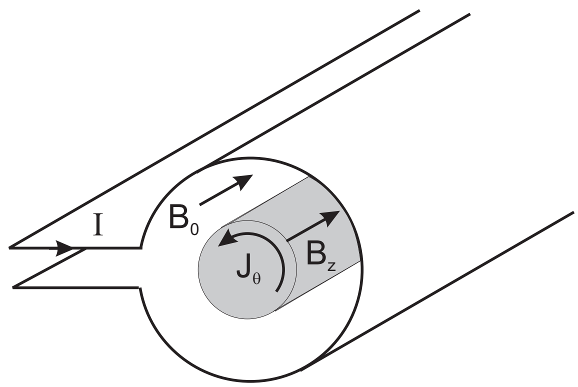

Figure 1: Theta pinch with a cylindrically symmetric plasma with pressure profile $p(r)$ and axial magnetic field $B_z(r)$. The axial vacuum magnetic field B_0 for $a < r < b$ is uniform.

a. Derive the one-dimensional equilibrium MHD force balance equation:

$$\nabla\left(p + \frac{B^2}{2\mu_0}\right) = \frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\mathbf{B}$$

and give an interpretation of each term.

b. Apply the above MHD force balance equation to the theta pinch to obtain an equation for $p(r)$, $B_z(r)$ and B_0.

c. Calculate the magnetic field inside the plasma $B_z(r)$.

d. Schematically draw the radial profiles $p(r)$, $B_z(r)$ and $J_\theta(r)$ for $0 < r < b$.

e. What can you say about the stability of the theta pinch? Motivate your answer.

PROBLEM 4 (25 points)

Consider the propagation of longitudinal electrostatic waves in a non-magnetized and collisionless plasma. Wave amplitudes are sufficiently small and the wave frequency sufficiently high so that the linear approximation can be used and the ion motion neglected. Also assume that the plasma temperature is constant and use the ideal gas law as the equation of state.

a. Write down first-order equations of continuity, momentum balance and Gauss' law.

b. Assume harmonically-varying perturbations of the form $e^{i(kz-\omega t)}$. Eliminate the electric field strength E_1 and the first-order electron velocity u_1 from these equations and derive the Bohm-Gross dispersion relation $\omega^2 = \omega_p^2 + v_{th}^2 k^2$ for electrostatic Langmuir waves. Calculate ω_p and v_{th}.

c. Sketch the Bohm-Gross dispersion relation $\omega(k)$ in a diagram and give a physical interpretation.

d. Discuss qualitatively what happens when you use a kinetic equation instead of fluid equations to calculate the dispersion relation of Langmuir waves.